Statistics

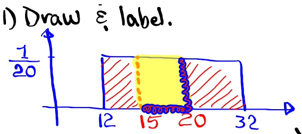
Winter 2022

Lecture 11

A Quick Review:

Consider a Uniform Prob. dist for all values from 12 to 32.

1) Draw & label.



2) Find $P(x=15) = \boxed{0}$

line has zero Area

3) Find $P(x<15 \text{ or } x>20)$

$= 1 - P(15<x<20) = 1 - (20-15)\cdot\frac{1}{20} = 1 - \frac{5}{20}$

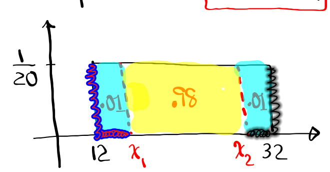
$= 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

Total Area = 1

4) Find two x-values that separate the middle 98% from the rest.

$1 - .98 = .02$

$.02 \div 2 = .01$



$(x_1 - 12)\cdot\frac{1}{20} = .01$

$x_1 - 12 = 20(.01)$

$x_1 - 12 = .2$

$\boxed{x_1 = 12.2}$

$(32 - x_2)\cdot\frac{1}{20} = .01$

$32 - x_2 = 20(.01)$

$32 - x_2 = .2$

$32 - .2 = x_2$

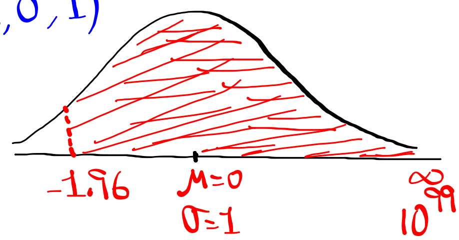
$\boxed{x_2 = 31.8}$

Find $P(Z > -1.96)$

$= \text{normalcdf}(-1.96, E99, 0, 1)$

(−)

$= \boxed{.975}$



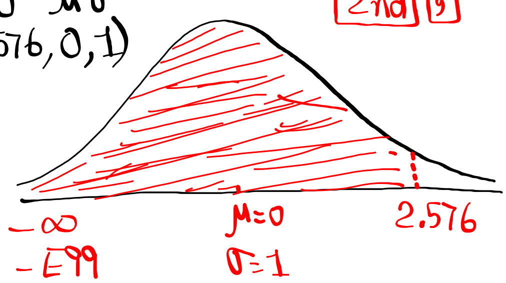
Find $P(Z < 2.576)$

L U μ σ

$= \text{normalcdf}(-E99, 2.576, 0, 1)$

(−) 2nd ,

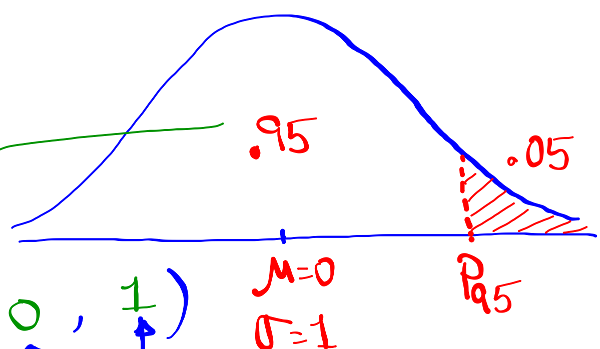
$= \boxed{.995}$



Find $Z = P_{95}$ Round to 3-decimal places

95% Below

5% above

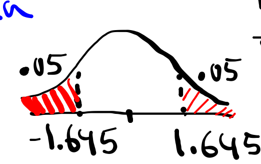


$Z = P_{95} = \text{invNorm}(.95, 0, 1)$

Left Area μ σ

$= \boxed{1.645}$

Now using Symmetry



$\boxed{P_5 = -1.645}$

Give $N(85, 8)$

Normal Prob. Dist

$\mu = 85$ $\quad \sigma = 8$

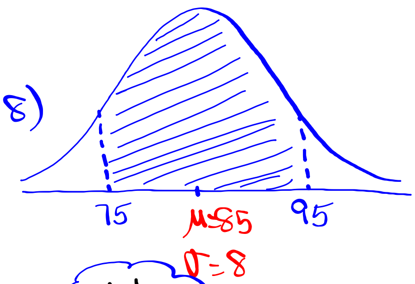
Find

$P(75 < x < 95)$

$= \text{normalcdf}(75, 95, 85, 8)$

$= \boxed{.789} \approx 78.9\%$

$\approx 79\%$



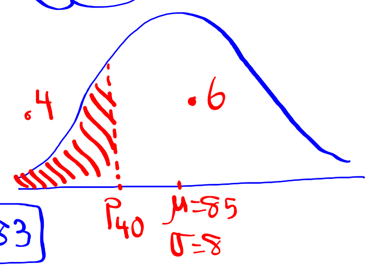
Find $x = P_{40}$, Round to a whole #

40% Below

60% above

$x = P_{40} = \text{invNorm}(.4, 85, 8)$

$= 82.973$ $\quad \boxed{x \approx 83}$

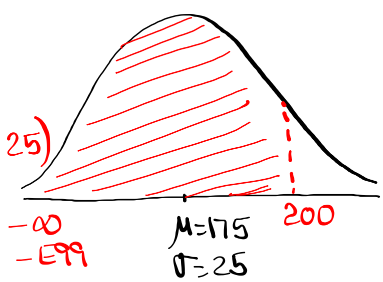


Consider a normal prob. dist with $\mu = 175$, and $\sigma = 25$.

Find $P(x < 200)$

$= \text{normalcdf}(-E99, 200, 175, 25)$

$= \boxed{.841}$



Find K Such that $P(x > K) = .025$

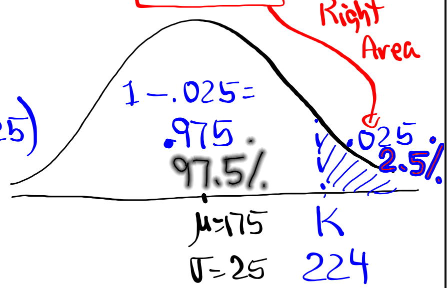
Right Area

Round to a whole #

$K = \text{invNorm}(.975, 175, 25)$

$= 223.999$

$\approx \boxed{224}$



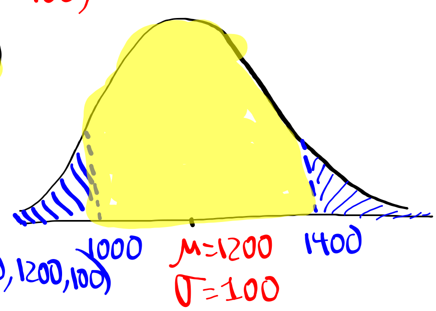
SAT Scores are normally dist. with $\mu = 1200$, and $\sigma = 100$

If we randomly select $\boxed{\text{one SAT}}^{x}$, find the prob. that it is below 1000 ($x < 1000$) or it is above 1400 ($x > 1400$).

$P(x < 1000 \text{ OR } x > 1400)$

$= 1 - P(1000 < x < 1400)$

↑ Total Area $= 1$



$= 1 - \text{normalcdf}(1000, 1400, 1200, 100)$

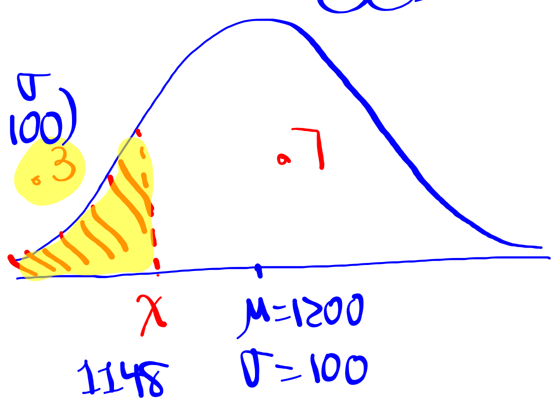
$= \boxed{.046} = 4.6\% \approx 5\%$

Find the SAT Score that separates the bottom 30% from the rest. Round to a whole.

$$x = P_{30} = \text{invNorm}(\underset{\text{Left Area}}{.3}, \underset{\mu}{1200}, \underset{\sigma}{100})$$

$= 1147.5599\ldots$

$\approx \boxed{1148}$



Clear all lists.

Store 2, 10, 18, and 26 in L1.

Use L1 to find

$\mu = 14$ $\sigma = 8.944$ $\sigma^2(\text{exact}) = 80$

Let's take all samples of size 2 with replacement

2,2	2,10	2,18	2,26
10,2	10,10	10,18	10,26
18,2	18,10	18,18	18,26
26,2	26,10	26,18	26,26

Find $\bar{x}$ of each sample

2	6	10	14
6	10	14	18
10	14	18	22
14	18	22	26

Find $\bar{x}$ of each sample

2	6	10	14
6	10	14	18
10	14	18	22
14	18	22	26

16 means

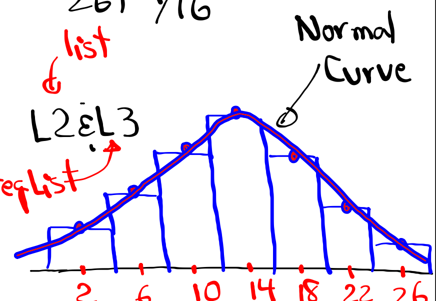
$\bar{x}$	$P(\bar{x})$
2	1/16
6	2/16
10	3/16
14	4/16
18	3/16
22	2/16
26	1/16

Prob. dist. of $\bar{x}$

list

$\bar{x} \to L2$, $P(\bar{x}) \to L3$, Use L2 & L3 with 1-Var Stats

Freq List



Prob. dist. Histogram

$\boxed{\mu = 14}$ $\sigma = 6.325$ $\boxed{\sigma^2(\text{exact}) = 40}$

$$\boxed{\mu_{\bar{x}} = \mu}$$ ✓

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$ ✓

Central Limit Theorem

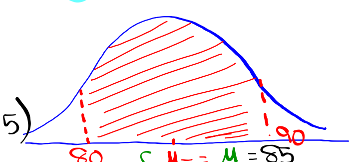
Suppose Scores on a math exam are $N(85,10)$ normally dist. with $\mu = 85$ and $\sigma = 10$.

If we randomly select 4 exams ($n=4$), find the prob. that their mean score ($\bar{x}$) is between 80 and 90.

$P(80 < \bar{x} < 90)$

$= \text{normalcdf}(80, 90, 85, 5)$

$= \boxed{.683}$



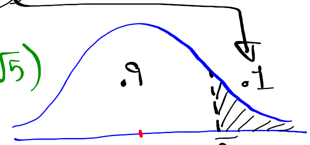
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 85 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{4}} = 5 \end{cases}$

Find $\bar{x}$ for randomly selected group of 5 exams that separates the top 10% from the rest.

$\bar{x} = \text{invNorm}(.9, 85, 10/\sqrt{5})$

$= 90.731$

≈ 91

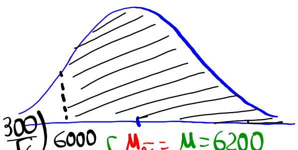


CLT $\begin{cases} \mu_{\bar{x}} = \mu = 85 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{5}} \end{cases}$

Salaries of nurses are normally dist. with $\mu = \$6200$ and $\sigma = \$300$. $N(6200, 300)$

If we randomly select 6 nurses ($n=6$), find the Prob. that their mean salary ($\bar{x}$) is above \$6000.

$P(\bar{x} > 6000)$



$= \text{normalcdf}(6000, E99, 6200, \frac{300}{\sqrt{6}})$

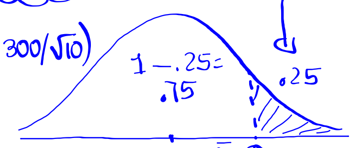
$= \boxed{.949}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 6200 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{300}{\sqrt{6}} \end{cases}$

Find $\bar{x}$ for randomly selected group of 10 nurses that separates the top 25% from the rest.

$\bar{x} = \text{invNorm}(.75, 6200, 300/\sqrt{10})$

$= \boxed{\$6264}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 6200 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{300}{\sqrt{10}} \end{cases}$

Speed of cars on certain freeway during certain time are normally distributed with mean speed of 74 mph with standard deviation of 12 mph. $N(74, 12)$

If we randomly select 5 cars, find the Prob. that their mean speed is

a) above 80

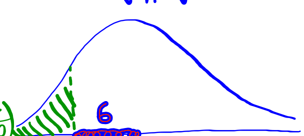
$P(\bar{x} > 80)$

$= \text{normalcdf}(80, E99, 74, 12/\sqrt{5})$

$= \boxed{.132}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 74 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{5}} \end{cases}$

b) below 68.



$= \text{normalcdf}(-E99, 68, 74, 12/\sqrt{5})$

$= \boxed{.132}$

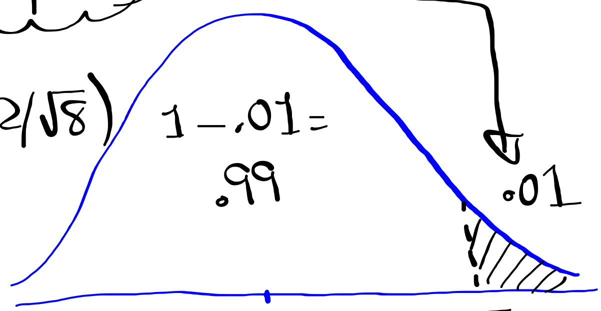
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 74 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{5}} \end{cases}$

For randomly selected 8 Cars, find their mean speed that separates the top 1% from the rest.

$\bar{x}$ = invNorm(.99, 74, $12/\sqrt{8}$)

= 83.869...

≈ $\boxed{84}$

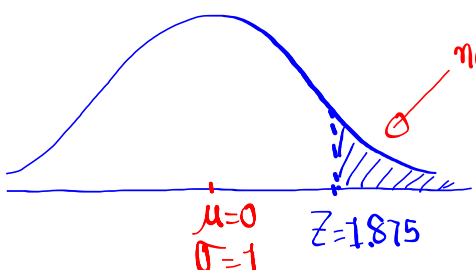


CLT $\begin{cases} \mu_{\bar{x}} = \mu = 74 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{12}{\sqrt{8}} \end{cases}$

$\boxed{\text{SG 18, 19, 20, and 21}}$

SG 22

Find the shaded area below



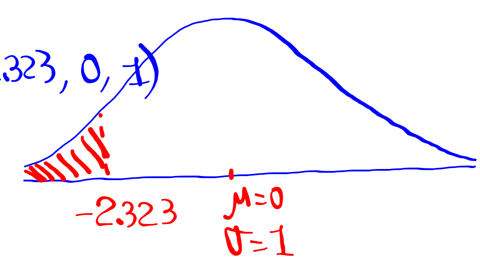
normalcdf(1.875, E99, 0, 1)

= $\boxed{.030}$

Find twice the area to the left of $z=-2.323$

2*normalcdf(−E99, −2.323, 0, 1)

= $\boxed{.020}$

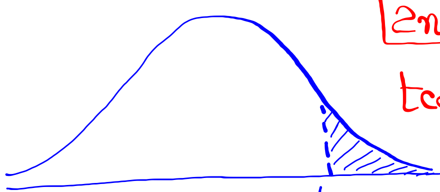


t - Dist , Student t - Dist.

- Bell-shape, Symmetric, Total Area = 1
- $\mu = 0$, σ unknown
- It comes with degrees of Freedom (df)
- we use tcdf or invT For this dist.

Find the shaded area below with df = 10.

$\boxed{2nd}\ \boxed{VARS}$



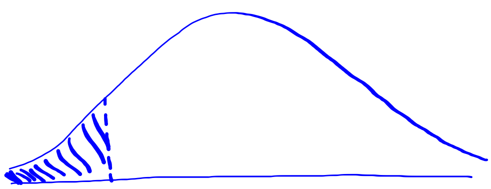
t = 2.025

tcdf(2.025 , E99 , 10)

lower upper df

$= \boxed{.035}$

Find the shaded area below with df = 15.



t = -1.567

tcdf(-E99, -1.567, 15) = $\boxed{.069}$ ✓

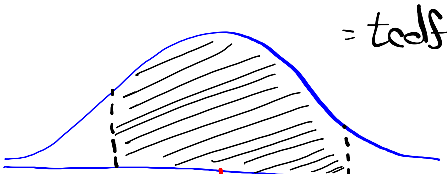
Lower Upper df

Find $P(-1.75 < t < 1.95)$ with df = 8

= tcdf(-1.75 , 1.95 , 8)

Lower Upper df

$= \boxed{.897}$



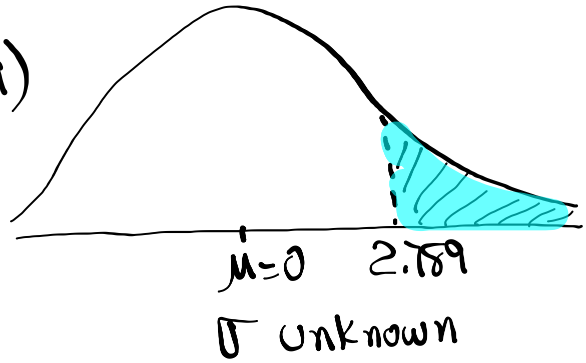
-1.75 $\mu = 0$ 1.95

σ unknown

Find twice the area to the right of $t = 2.789$ with $df = 9$.

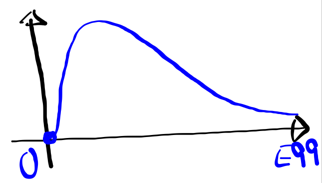
$2 * \text{tcdf}(2.789, E99, 9)$

$= \boxed{.021}$

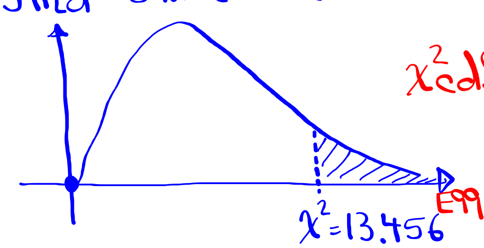


Chi-square dist: $\underbrace{\text{Chi-square}}_{\chi^2}$

- Graph begins at 0, skewed to the right
- Not symmetric, Total Area = 1
- It comes with df.
- use χ^2cdf for this one.



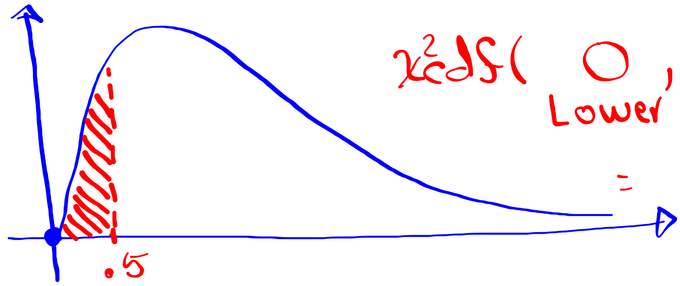
Find shaded area below with $df = 7$.



$\chi^2\text{cdf}(\underset{\text{Lower}}{13.456}, \underset{\text{upper}}{E99}, \underset{df}{7})$

$= \boxed{.062}$ ✓

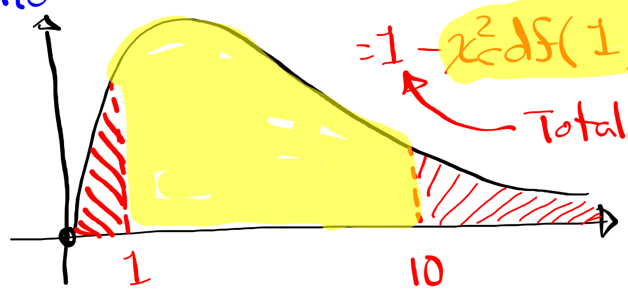
Find $P(\chi^2 < .5)$ with $df=8$.



$\chi^2cdf(\underset{\text{Lower}}{0}, \underset{\text{upper}}{.5}, \underset{df}{8})$

$= \boxed{1.3 \times 10^{-4}}$

Find $P(\chi^2 < 1$ OR $\chi^2 > 10)$ with $df=5$.



$= 1 - \chi^2cdf(1, 10, 5)$

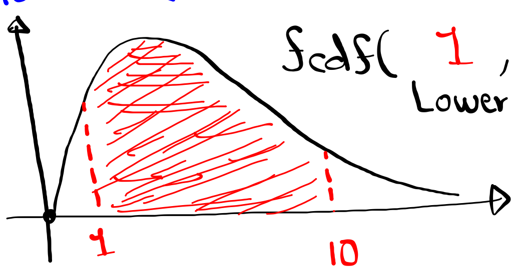
Total Area

$= \boxed{.113}$

F - Dist

- Graph is similar to χ^2-Dist.
- It comes with two degrees of freedom:
  - Numdf = Ndf
  - Denodf = Ddf
- use fcdf for this one

Find $P(1 < F < 10)$ with $Ndf=4$, $Ddf=20$.

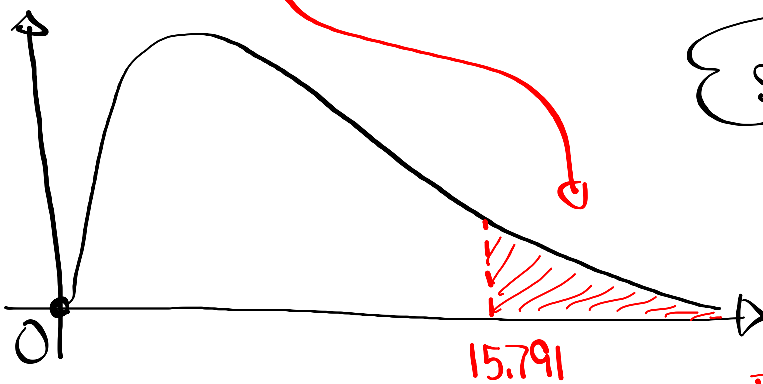


$fcdf(\underset{\text{Lower}}{1}, \underset{\text{Upper}}{10}, \underset{Ndf}{4}, \underset{Ddf}{20})$

$\boxed{.431}$

Find $P(F > 15.791)$ with Ndf=5 & Ddf=30

SG 22 ✓



$Fcdf(15.791, E99, 5, 30) = \boxed{1.2 \times 10^{-7}}$

Live QZ Extra Credit

Consider a binomial Prob. dist with $n=400$, and $p=.5$

1) P(exactly 195 successes)
$= P(x=195) = binompdf(400, .5, 195) = \boxed{.035}$

2) P(at most 210 successes)
$= P(x \le 210) = binomcdf(400, .5, 210) = \boxed{.853}$

3) P(at least 190 successes) $= \boxed{.853}$
$= P(x \ge 190) = 1 - P(x \le 189) = 1 - binomcdf(400, .5, 189)$